

## Anti-Aliasing Filter, Group Delay and Phase Match

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### ***Anti-Aliasing Filter and the Use of Sigma-Delta Converters***

The anti-aliasing filter is an important part in the dynamic signal analyzer input front end. It is used to filter out unwanted high frequency content before the analog signal is converted to the digital domain. Without an anti-aliasing filtering, high frequency noise is aliased into lower frequency spectrum during the digitization process which can lead to misinterpretation of the data. In the old days, the anti-aliasing filters were realized purely in the analog domain. Typically, a 6 or 8<sup>th</sup>-order Butterworth filter was used in order to achieve reasonable attenuation of high frequency noise.

The important parameters of an anti-aliasing filter include the cutoff frequency, flatness of the filter passband, attenuation of its stopband, and the linearity of its phase in the useful frequency range.

The drawbacks of analog anti-aliasing filter are several folds:

- (1). The  $f_a/f_s$  ratio is low, where  $f_a$  is the analyzer cutoff frequency and  $f_s$  is the sampling frequency. Usually the value of  $f_a/f_s$  is about 0.2 to 0.3. This required that the instruments needed to sample at 80 kHz or higher in order to get a useful frequency up to 20 kHz.
- (2). The phase characteristics of analog filters are not consistent therefore the phase match between each input channels are not good. It can range from a few degrees to as much as ten degrees or more up to 20 kHz.
- (3). The phase of analog filters is not linear. A none-linear phase in a filter will result in distortion in the time domain. For example an impulse signal passing through a filter will have a very different peak reading and changed shape in time domain.
- (4). The attenuation of an analog low pass filter is usually in the magnitude of -65 dB or less. Therefore the anti-aliasing effect is poor. When the anti-aliasing effect is not good, the system dynamic range and total harmonic distortion specs will suffer.

A revolution started in the mid nineties when the semiconductor industry introduced a new technology, the sigma delta A/D converter. In 1994, Crystal Instruments was among the very first companies in the world using the sigma-delta A/D converter in the dynamic signal analyzer. This innovation eliminated the need of large size analog filters and made it possible to put a four channel signal analyzers into a package the size of a PCMCIA card while realizing significant improvements in the anti-aliasing filter performance.

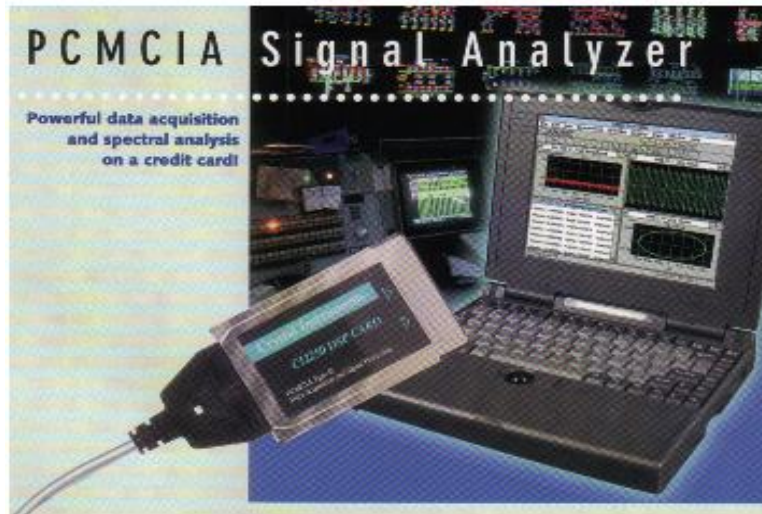


Figure 1 The World's Smallest Dynamic Signal Analyzer released in 1995. It is probably the first product using sigma-delta converters in this industry.

Nowadays, 100% of the modern dynamic signal measurement instruments use sigma-delta A/D converters.

A typical input channel of the dynamic measurement instrument has the following building blocks:

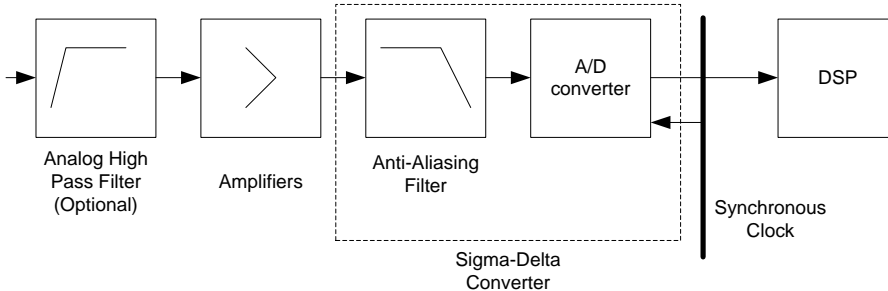


Figure 2 Building blocks of an input channel in a dynamic signal analyzer.

The anti-aliasing filter realized in modern instruments is part of the sigma-delta converter circuit. Most often it is simply a low-order low pass-filter with a cutoff frequency at a few Mega-Hertz. The sigma-delta A/D converter is sampled at very high frequency, typically 10 MHz. Then it uses a digital decimation technique to down-sample the signal down to the twenty kilohertz range or below. This implementation in the combined analog and digital domain creates an ideal anti-aliasing capability so that all the drawbacks of the old analog anti-aliasing filters are eliminated. The magnitude vs. frequency of a typical anti-aliasing filter is shown below:

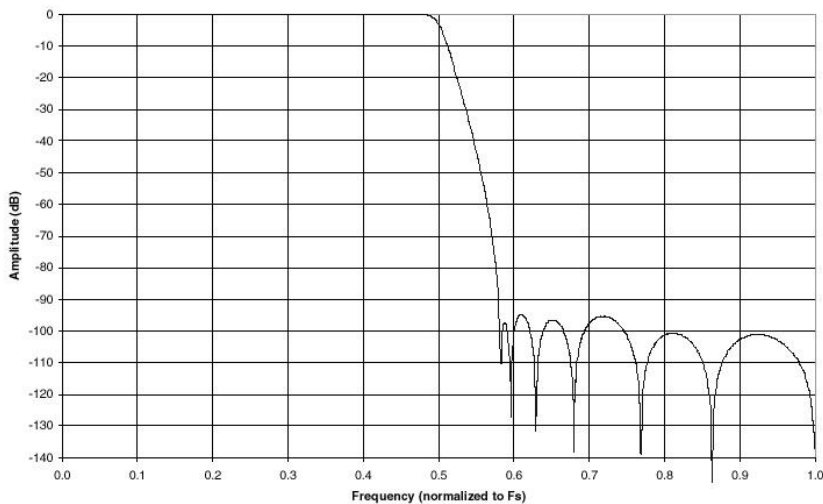


Figure 3. Frequency response of a typical digital anti-aliasing filter.

The advantages of using the sigma-delta converter and such new anti-aliasing technique are many, including:

- (1). The packaging size of the front-end is much smaller.
- (2). The  $f_a/f_s$  ratio is higher, therefore the useful frequency is higher for the same sampling rate. In CI's product, the  $f_a/f_s$  ratio is more than 0.45. This means that with an FFT of 1024 points, the useful frequency line number is over 450 lines.
- (3). The filter attenuation can be 100 dB or higher. The system dynamic range and THD can be much higher than before.
- (4). The filter phase is near-linear therefore the group delay is near-constant.
- (5). Cross channel performance can achieve extremely high phase match, with less than 1 degree up to 20kHz. In the old days this was unimaginable.

### **Group Delay and Phase Match**

Group delay of the filter is a measure of the transit time of a signal through the filter versus frequency. Group delay is a useful measure of phase distortion, and is calculated by differentiating the phase response of the filter versus frequency. Another way to say this is that group delay is a measure of the slope of the phase spectrum.

The variations in group delay cause signal distortion, just as deviations from linear phase cause distortion. Group delay is just another way to look at linear phase distortion. A filter with constant group delay must have a linear phase.

Mathematically, if the frequency transfer-function of a filter is represented by  $H(\omega)$ .

$$H(\omega) = M(\omega) e^{j\Phi(\omega)}$$

where  $\Phi(\omega)$  is the phase spectrum. Then the derivative of the phase spectrum with respect to frequency,  $-\frac{\Delta\Phi(\omega)}{\Delta\omega}$  is the so called group delay.

Usually, in the dynamic signal analyzer the group delay, phase-linearity and time delay of the A/D converters of different channels can be measured by one single value: **phase match**. Phase match is the maximum phase deviation between each pair of input channels at a certain frequency. A good instrument should claim the phase match as no more than a few degrees at their useful frequency.

Phase match also reflects the difference of the time delays between each pair of input channels. For example, the figure below shows the phase spectrum measured from two input channels with simultaneous sampling while both channels are fed the same random excitation signal. Phase spectrum is the phase associated with the cross-spectrum between the two channels.

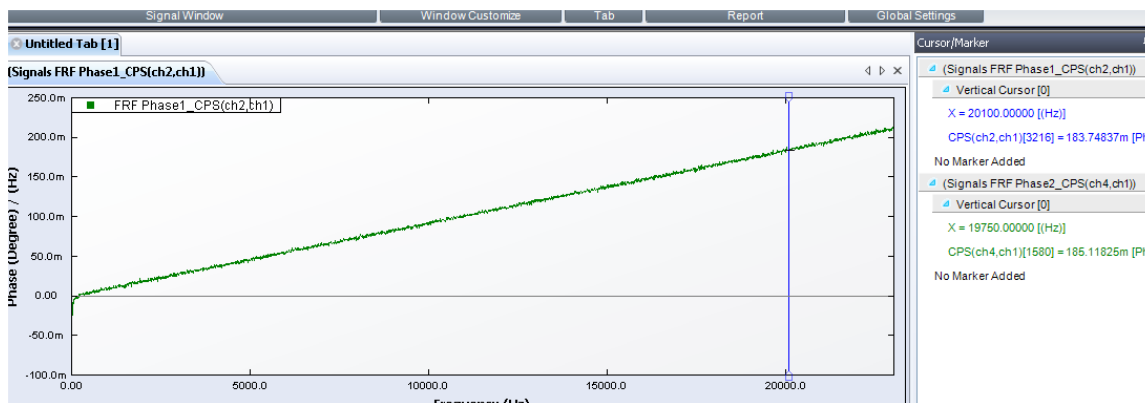


Figure 4 Phase spectrum between two input channels, measured using cross-spectrum technique.

The phase spectrum shows perfect linearity. This means that the time delay of two input channels is a constant. Let's pick up the phase value at any frequency, say 20,200 Hz, and calculate the time delay. The phase is 0.187 degrees and the time delay is calculated as:

$$\text{Time delay} = (1/20200\text{Hz}) * (0.187 \text{ degree}/360 \text{ degree}) = (2.57\text{E-}8) \text{ second} = 25.7\text{ns}$$

The 25.7 nanoseconds is the tiny time delay between these two specific input channels. Such an extremely small time delay was never possible with analog filter technology.

This delay (albeit very small) is the result of several factors, including:

- (1). A slight variation of the synchronous clock signals going to each A/D converters shown in Fig 2.
- (2). A slight variation of the group delay of the anti-aliasing filter of each channel.

Considering that the instrument is measuring signals mostly below 20 kHz, a time delay of a few nanoseconds is more than acceptable.

While sigma-delta converters have tremendous advantages over using traditional analog anti-aliasing filters, care must be taken when following applications are involved:

1. In transient data capture, the sharp brick-wall type of anti-aliasing filter of sigma delta converter will create strong ripple effect to the time signal. The waveform will be distorted without extra treatment. Fortunately this problem can easily be resolved by applying another digital filter in the DSP that can accurately preserve the time signal shape. This is done in all CI products.
2. The sigma-delta A/D converter has a longer time delay than those made with other technologies such as flash CMOS A/D. The delay can be as long as several milliseconds. This drawback makes sigma-delta converter less usable in motion control applications where absolute response time of the converter is critical.

In conclusion, the application of sigma-delta A/D converters in anti-aliasing filters has revolutionized dynamic signal analyzer products. They allow products to be miniaturized while producing marked improvements in performance such as channel match and maximum frequency. While nearly all manufacturers now use this technology, Crystal Instruments was among the first developers of this technology and has produced many related innovations that are included in all their products.