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Crystal Instruments Webinar Series

# Basics of FFT Signal Analysis

Hosted by Andrew Snyder

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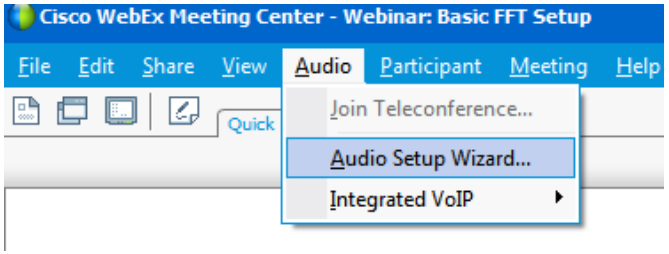

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# CI Webinar Schedule

| Date | Topic |
|---|---|
| August 18 | Basic FFT setup |
| September 15 | Routes for data measurement |
| October 20 | Impact testing |
| November 17 | Sound Level Measurements |
| December 15 | Order Tracking |

See www.go-ci.com/webinars.html for up-to-date schedule

All Webinars held at 11am AND 8pm, Pacific Time

Hosted from the CI headquarters in Santa Clara, CA

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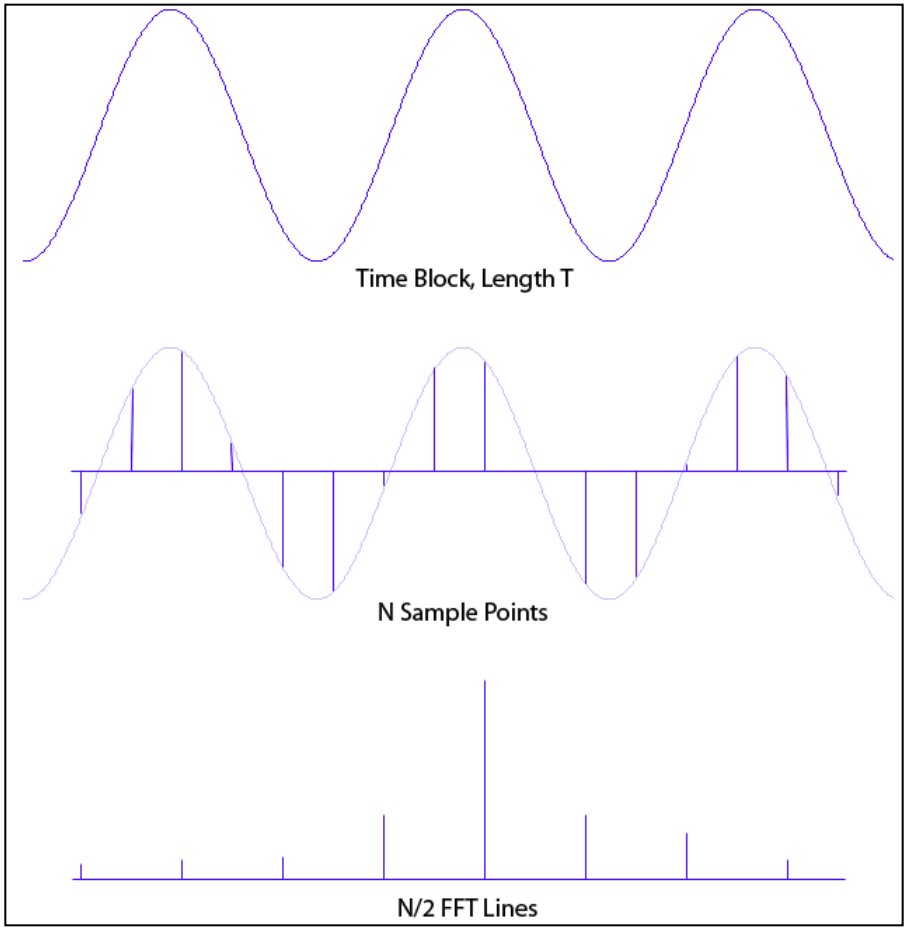
# Basics of FFT Signal Analysis



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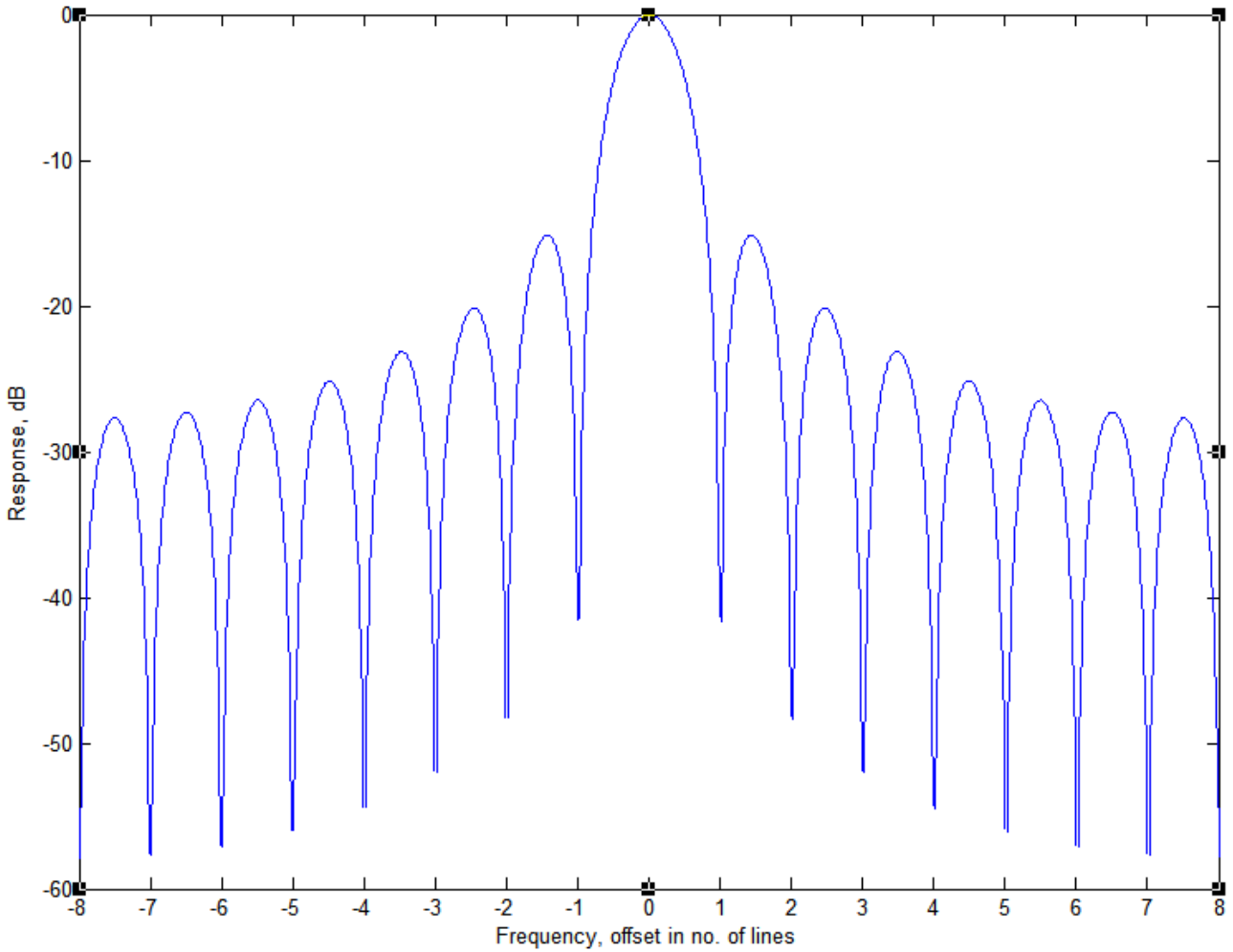
# FFT Line Response



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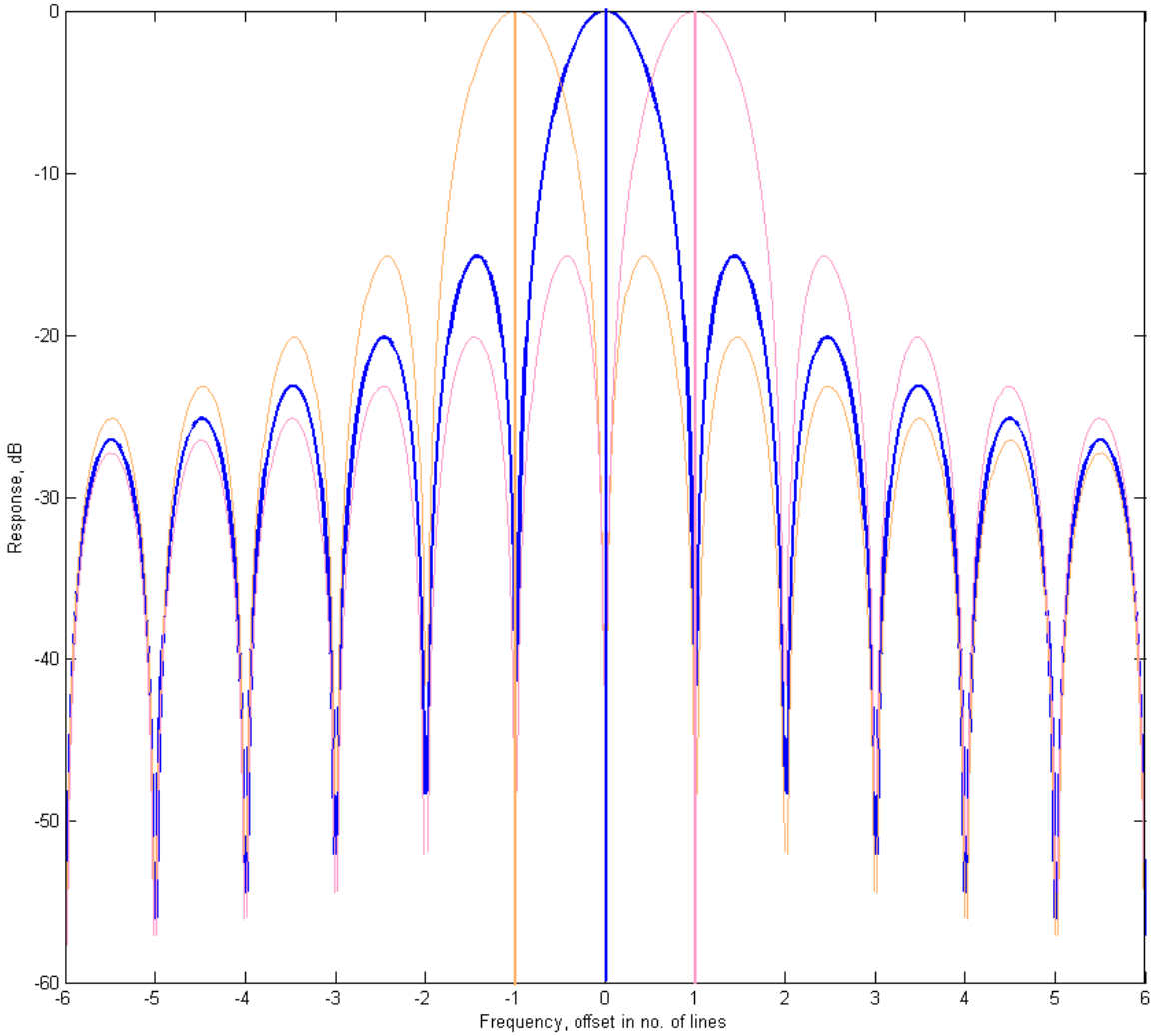
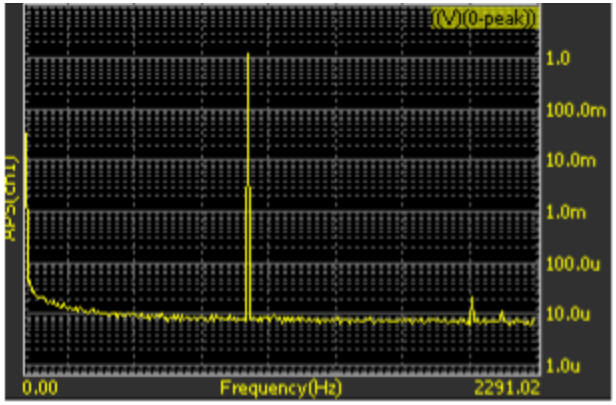
# FFT Line Response

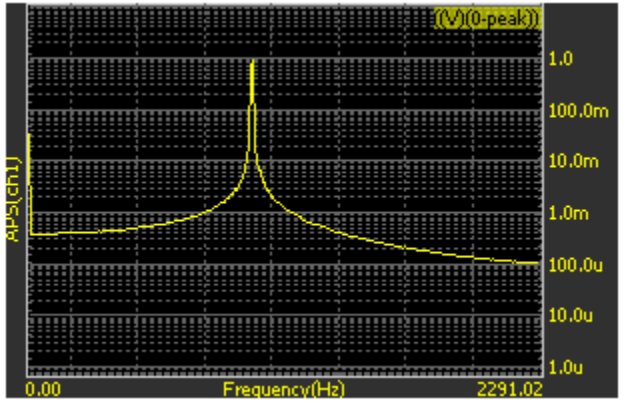


# Leakage

Frequency centered on FFT line




Frequency between FFT lines



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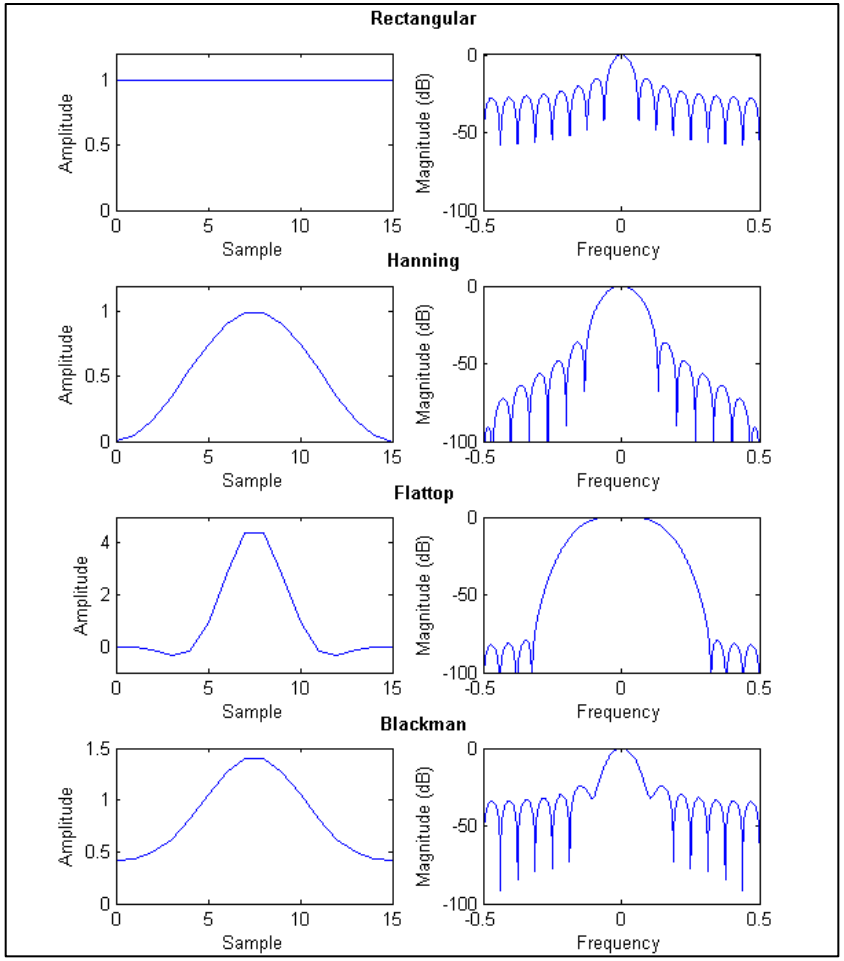
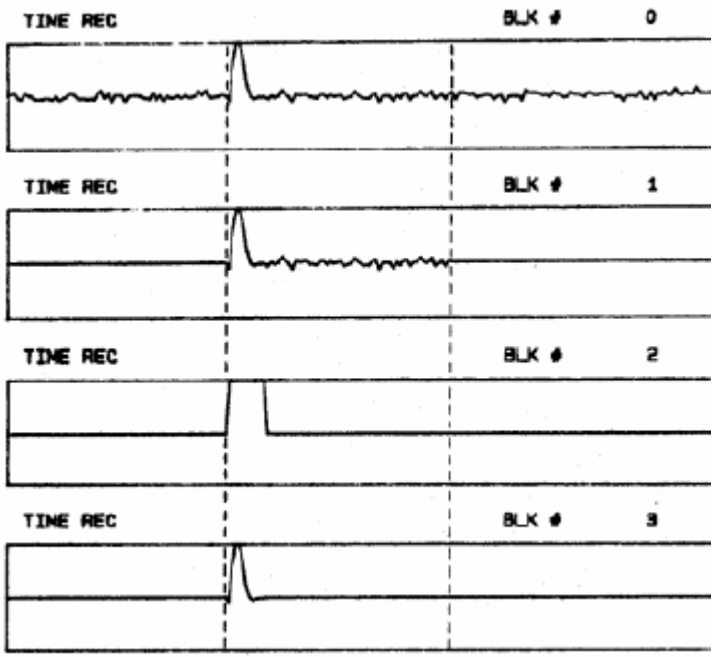
# Windowing Functions

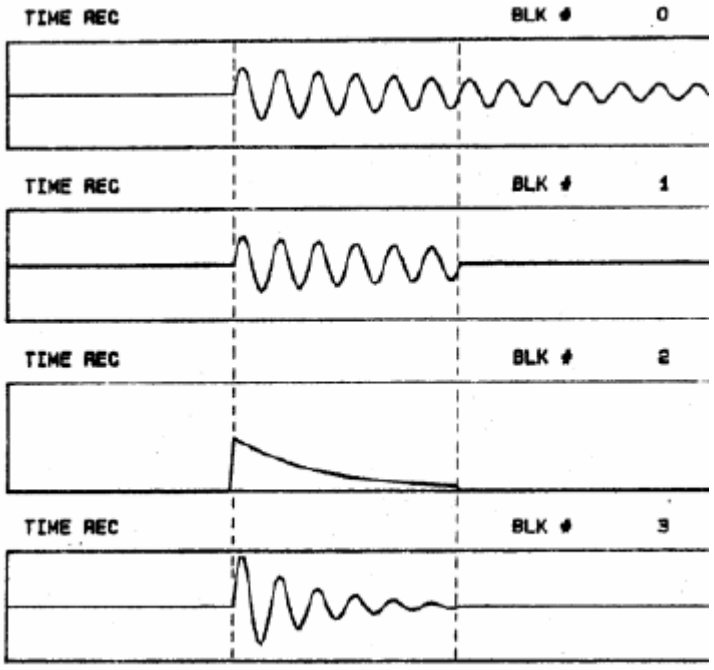


# Force and Exponential Windows

Force window: excitation




Exponential window: response




Pictures courtesy of University of Massachusetts, Lowell

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# Analysis Parameters

- Sample rate $F_s$
- Block size $N$
- Windowing function
- Averaging
- Overlap

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# Resolution and Range

- FFT takes $N$ sample points and returns $\frac{N}{2}$ frequency lines
- Some higher-frequency lines are lost because of the *anti-aliasing filter*
- Highest frequency controlled by **sampling rate**
- Lowest frequency controlled by **block size**

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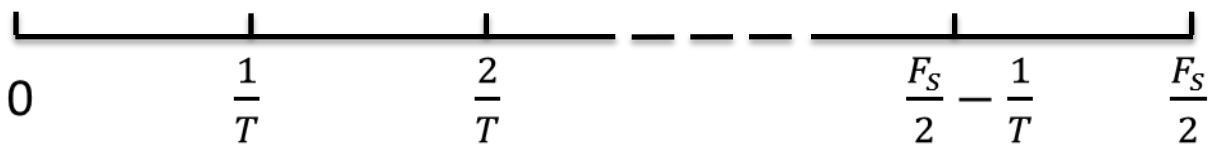
# Resolution and Range

- Highest frequency $F_a = \frac{F_s}{2}$ (Nyquist theorem)
- Lowest frequency = $\frac{1}{T}$ *(block fundamental frequency)*
- Lines evenly spaced in between. Spacing = $\frac{1}{T}$




**Frequency Lines**

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# Resolution and Range

- Highest frequency $F_a = \frac{F_s}{2}$
- Lowest frequency = $\frac{1}{T}$
- Lines evenly spaced in between. Spacing = $\frac{1}{T}$
- Sampling rate is time *resolution*, block length is time *range*

**Range in one domain ↔ Resolution in other domain**

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# Spectrum Type and Scaling

- FFT returns *complex-valued* amplitudes
- Real part represents cosine components, imaginary part represents sine components (90° phase difference)
- Can be converted to *magnitude* and *phase*
- Squared magnitude represents **signal power**

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# Spectrum Types

- Spectrum Types:
  - Magnitude: **Amplitude Spectrum**
  - Squared magnitude: **Power Spectrum**
  - Squared magnitude per unit bandwidth: **Power Spectral Density**
  - Squared magnitude × block time length: **Energy Spectrum**
  - Squared magnitude × block length per unit bandwidth: **Energy Spectral Density**

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# Spectrum Types

- Periodic signals (discrete frequencies): Amplitude or Power Spectrum
- Broadband random signals: Power Spectral Density
- Transient Signals: Energy Spectral Density

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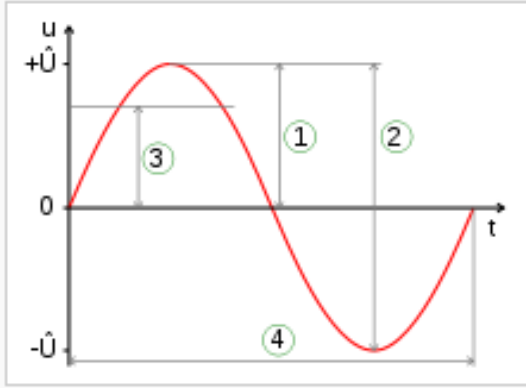
# Amplitude Spectrum Scaling

- Magnitude is peak value
- 2·peak = peak-to-peak
- $\frac{1}{\sqrt{2}}$·peak = Root-Mean-Square (RMS)
  - Square of RMS value proportional to power

From Wikipedia:




A sinusoidal curve
1 = Peak amplitude ($\hat{U}$),
2 = Peak-to-peak amplitude ($2\hat{U}$),
3 = RMS amplitude ($\hat{U}/\sqrt{2}$),
4 = Wave period (not an amplitude)

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# Averaging

- Combine multiple time blocks together to form one spectral estimate
- Random data: higher average number, better estimate of random characteristics

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# Averaging Types

- Average can be of different types of frequency data
- Complex frequency data $S_x$:
  Linear Spectrum Average $G_x$
  - Averages both magnitude and phase
  - Random data goes to 0 as average number increases
- Power spectrum $S_{xx}$:
  Power Spectrum Average $G_{xx}$
  - No phase data
  - Random data goes to average power level as average number increases

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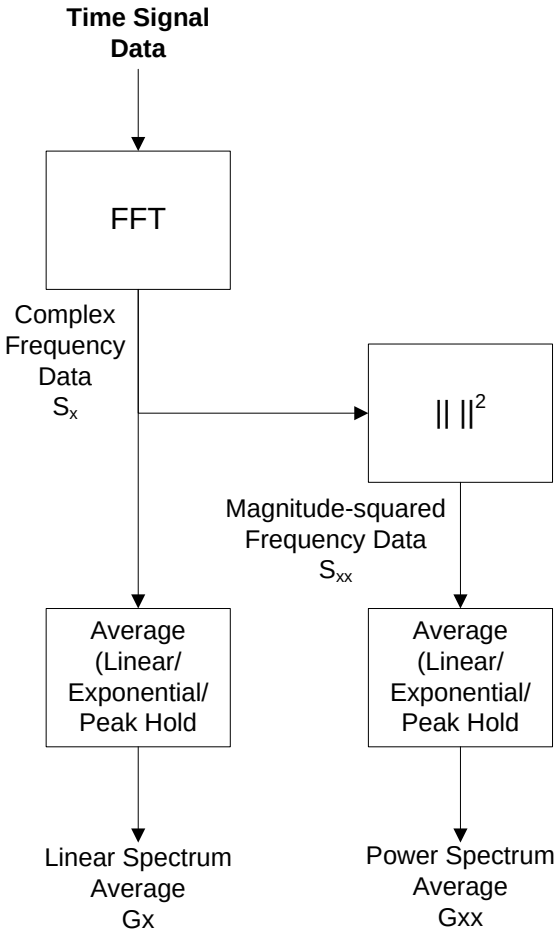
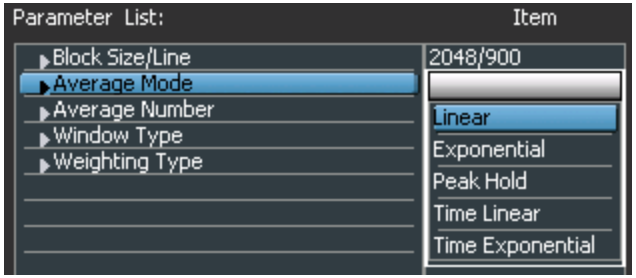
# Averaging Methods

- Linear: $G_{xx} = \frac{S_{xx}^1 + S_{xx}^2 + \cdots + S_{xx}^n}{n}$
- Exponential: $G_{xx}^{n+1} = \alpha S_{xx} + (1 - \alpha) G_{xx}^n$
- Peak-Hold: $G_{xx}(k) = \max_m S_{xx}^m(k)$

# Averaging




**In CoCo:**

- Linear Spectrum:
  - Time Linear
  - Time Exponential
- Power Spectrum:
  - Linear
  - Exponential
  - Peak-Hold

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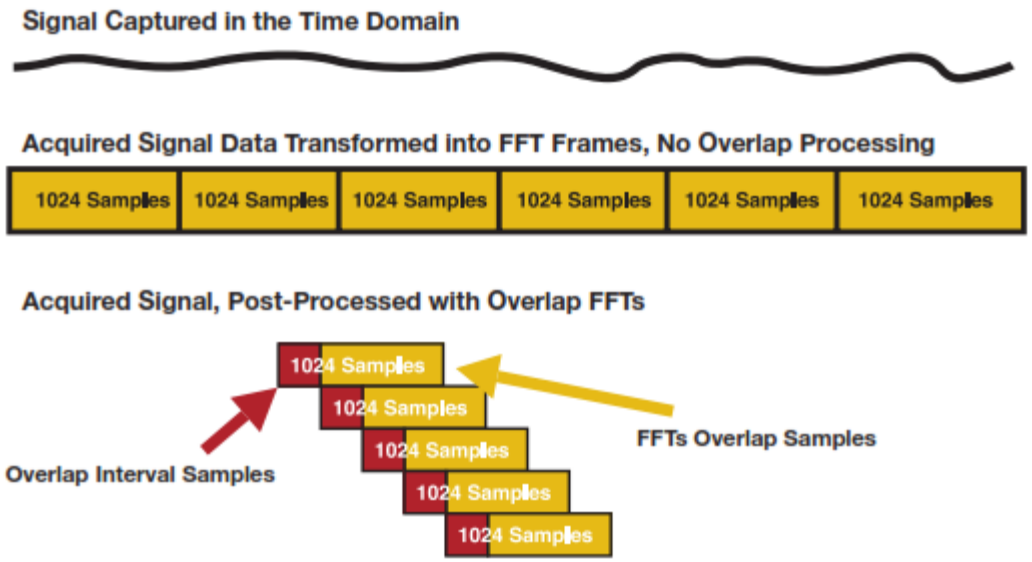
# Overlap Processing

- A proportion of each time frame can be “reused” in subsequent frames
- Faster updating
- Better averaging in same time period, only up to 50%




Graphic from *Understanding FFT Overlap Processing*, Tektronix Corporation

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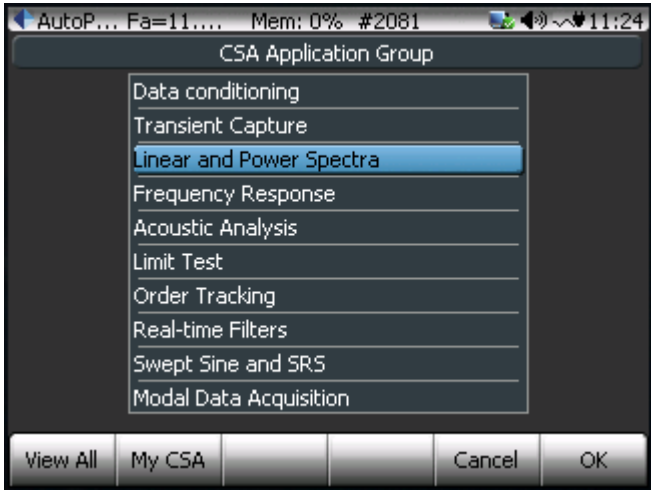
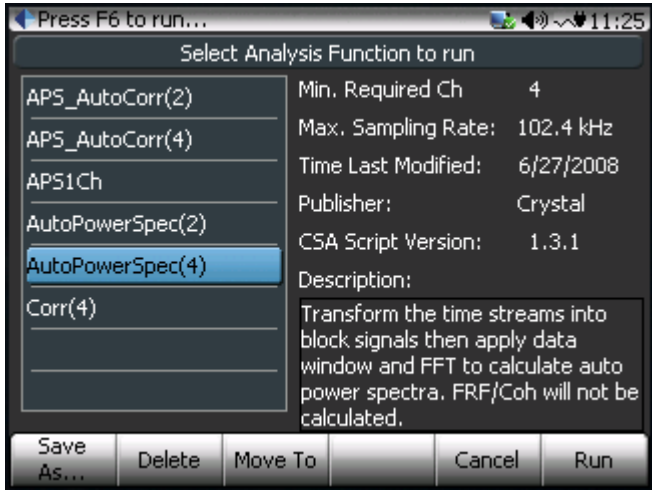
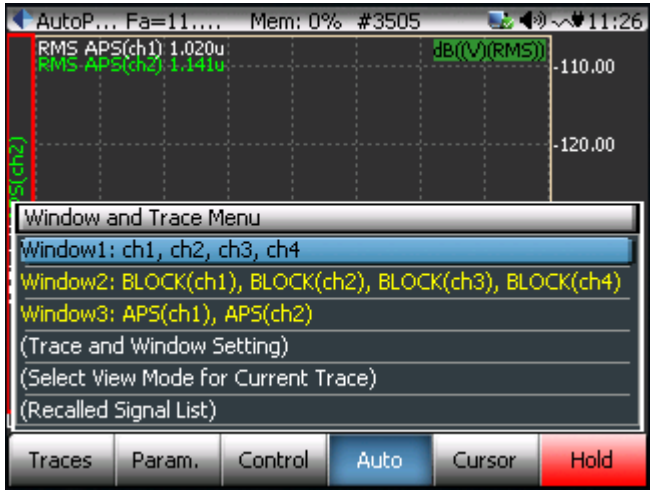
# CoCo Operation

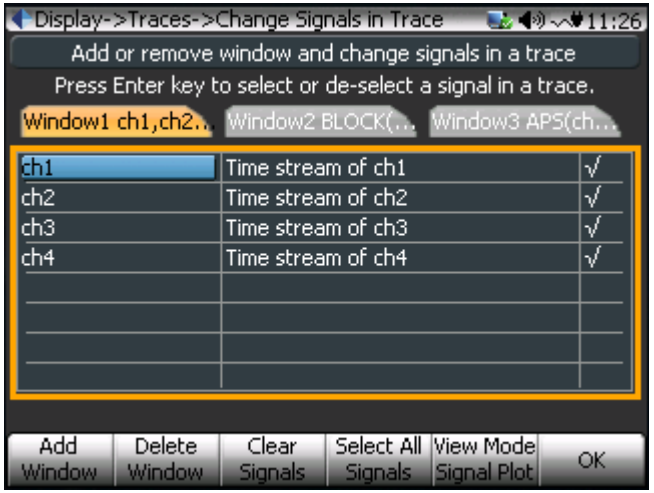
# CSAs


AutoP... Fa=11.... Mem: 0% #2081 11:24
CSA Application Group
Data conditioning
Transient Capture
Linear and Power Spectra
Frequency Response
Acoustic Analysis
Limit Test
Order Tracking
Real-time Filters
Swept Sine and SRS
Modal Data Acquisition
View All
My CSA
Cancel
OK

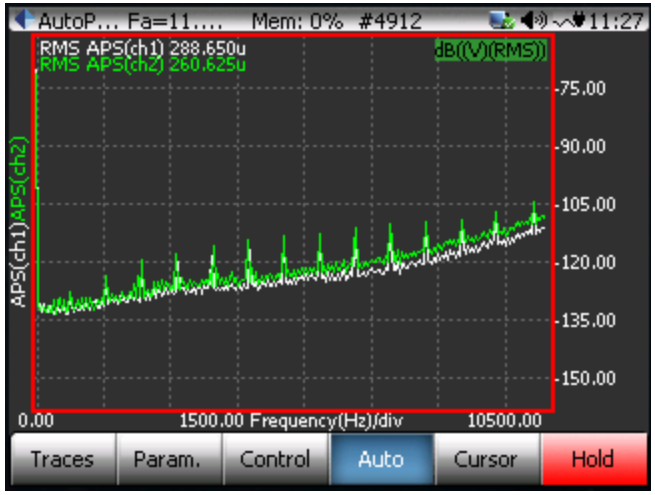

Press F6 to run... 11:25
Select Analysis Function to run
APS_AutoCorr(2)
APS_AutoCorr(4)
APS1Ch
AutoPowerSpec(2)
AutoPowerSpec(4)
Corr(4)
Min. Required Ch 4
Max. Sampling Rate: 102.4 kHz
Time Last Modified: 6/27/2008
Publisher: Crystal
CSA Script Version: 1.3.1
Description:
Transform the time streams into block signals then apply data window and FFT to calculate auto power spectra. FRF/Coh will not be calculated.
Save As...
Delete
Move To
Cancel
Run

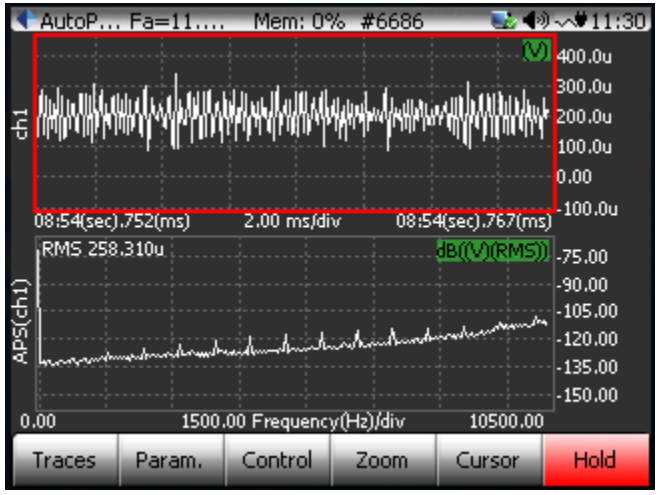
# Windows and Traces



# Signal Display



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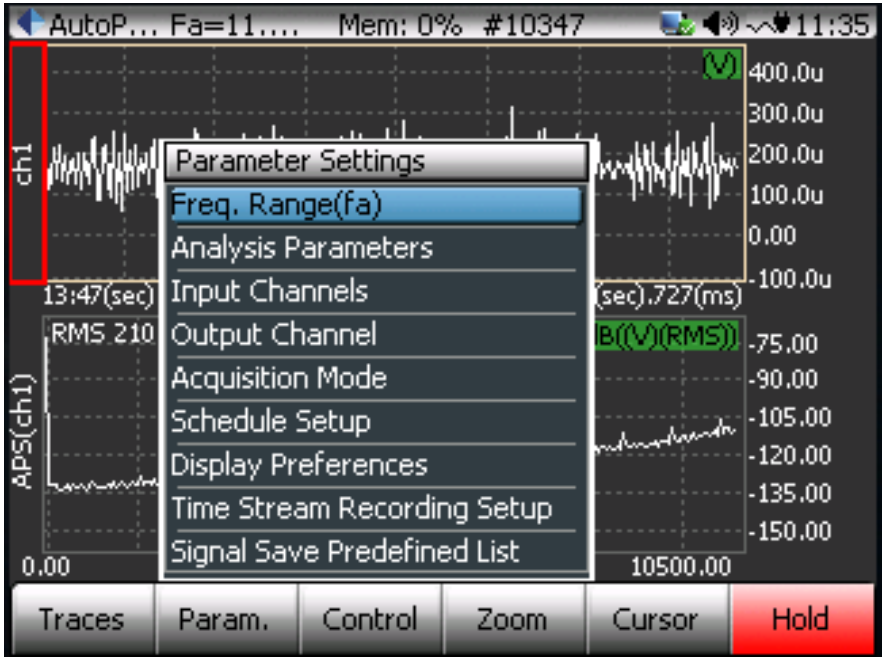
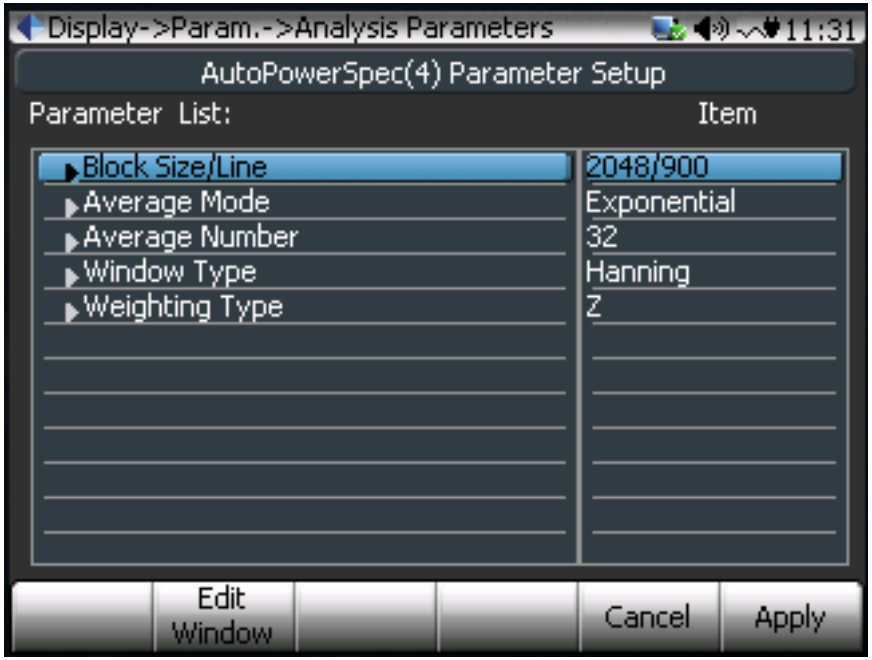
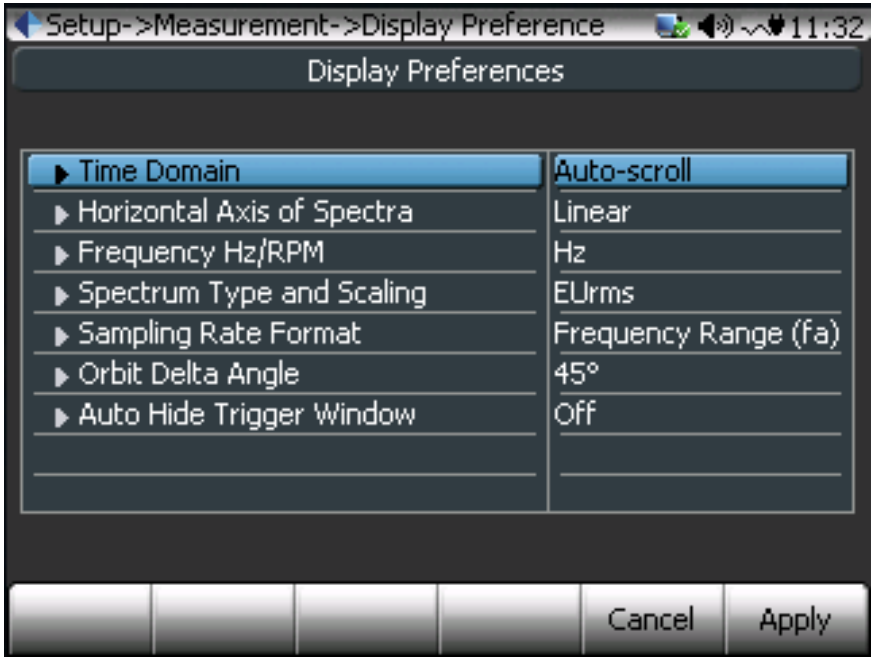
# Param. Menu



# Analysis Parameters

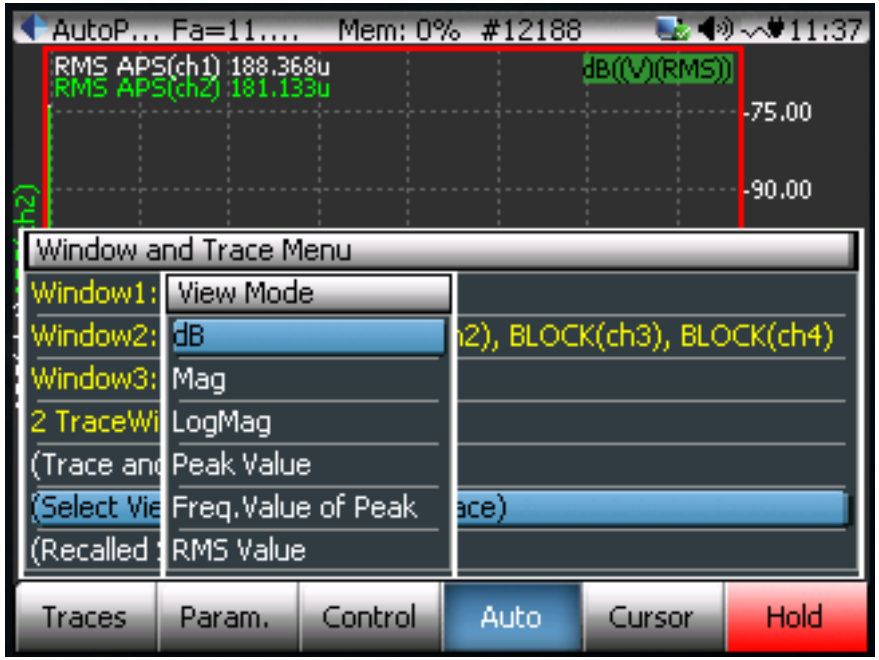


# Display Parameters



# dB, Mag, Log

$$\mathrm{dB} = 10 \log \frac{P}{P_{ref}} = 20 \log \frac{V}{V_{ref}}$$



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# Brought to you by the Crystal Instruments Team

James Zhuge, President

Tim Hsiao

John Holler

Darren Fraser

Justin Tang

Yao Liang

Andrew Snyder

Mike Mu

**Crystal Instruments Corporation**
4699 Old Ironsides Dr., Ste. 100
Santa Clara, CA 95054, USA

Phone: 408-986-8880
**Visit us online at http://www.go-ci.com**